

# On the $\text{SLq}(2)$ extension of the standard model and the measure of charge

Robert J. Finkelstein

*Physics & Astronomy, University of California, Los Angeles,  
475 Portola Plaza, Los Angeles, California 90095, USA  
finkel@physics.ucla.edu*

## Abstract

Our  $\text{SLq}(2)$  extension of the standard model is constructed by replacing the elementary field operators,  $\Psi(x)$ , of the standard model by  $\Psi_{mm'}^j(x)D_{mm'}^j$ , where  $D_{mm'}^j$  is an element of the  $2j + 1$  dimensional representation of the  $\text{SLq}(2)$  algebra, which is also the knot algebra. The allowed quantum states  $(j, m, m')$  are restricted by the topological conditions

$$(j, m, m') = \frac{1}{2}(N, w, r + o)$$

postulated between the states of the quantum knot  $(j, m, m')$  and the corresponding classical knot  $(N, w, r + o)$  where the  $(N, w, r)$  are (the number of crossings, the writhe, the rotation) of the 2d projection of the corresponding oriented classical knot. Here  $o$  is an odd number that is required by the difference in parity between  $w$  and  $r$ . There is also the empirical restriction on the allowed states

$$(j, m, m') = 3(t, -t_3, -t_0)_L$$

that holds at the  $j = \frac{3}{2}$  level, connecting quantum trefoils  $(\frac{3}{2}, m, m')$  with leptons and quarks  $(\frac{1}{2}, -t_3, -t_0)_L$ . The so constructed knotted leptons and quarks turn out to be composed of three  $j = \frac{1}{2}$  particles which unexpectedly agree with the preon models of Harrari and Shupe. The  $j = 0$  particles, being electroweak neutral, are dark and plausibly greatly outnumber the quarks and leptons. The  $\text{SLq}(2)$  or  $(j, m, m')$  measure of charge has a direct physical interpretation since  $2j$  is the total number of preonic charges while  $2m$  and  $2m'$  are the numbers of writhe and rotation sources of preonic charge. The total  $\text{SLq}(2)$  charge of a particle, measured by writhe and rotation and composed of preons, sums the signs of the counterclockwise turns (+1) and clockwise turns (-1) that any energy-momentum current makes in going once around the knot. In this way the handedness of the knot reduces charge to a geometric concept similar to the way that curvature of spacetime encodes mass and energy. According to this model, the leptons and quarks are  $j = \frac{3}{2}$  particles, the preons are  $j = \frac{1}{2}$  particles, and the  $j = 0$  particles are candidates for dark matter.

# 1 Introduction<sup>(1)(2)(3)</sup>

In our SLq(2) extension of the standard model the field operators  $\Psi(x)$  of the elementary fermions are replaced in the following way:

$$\Psi(x) \rightarrow \hat{\Psi}_{mm'}^j(x) D_{mm'}^j \quad (1.1)$$

where  $D_{mm'}^j$  is an element of the  $2j + 1$  dimensional representation of the SLq(2) algebra and where the  $\hat{\Psi}_{mm'}^j(x)$  satisfy the Lagrangian of the standard model after modification by the form factors generated by the  $D_{mm'}^j$ . Since SLq(2) describes the symmetry of the classical knot, the field quanta, lying in the SLq(2) algebra, may be characterized as knotted or may be described as quantum knots.

We further define the quantum knot by postulating the following kinematical restriction to allowed states

$$(j, m, m') = \frac{1}{2}(N, w, r + o) \quad (1.2)$$

where  $N$  is the number of crossings,  $w$  is the writhe,  $r$  is the rotation of the 2d projection of the corresponding oriented classical knot; and  $o$  is an odd integer that is required since  $w$  and  $r$  are of opposite parity. Here  $o$  is a new quantum number which we set at  $o = 1$  for the simplest knot, the quantum trefoil, and may have other values for other knots. We also introduce “the quantum rotation”  $\tilde{r} \equiv r + o$  where  $o$  appears as a zero-point rotation of the quantum knot. Equation (1.2), in addition to defining the kinematics of the quantum knot, establishes a correspondence between quantum knots  $(j, m, m')$  and 2d-projections of oriented classical knots  $(N, w, r)$ .

The knot picture of the elementary particles is more attractive if the simplest particles are the simplest knots. We therefore consider the possibility that the most elementary fermions with electroweak isotopic spin  $t = \frac{1}{2}$  are the most elementary quantum knots, the quantum trefoils with  $N = 3$  and  $o = 1$ . This possibility is supported by the following empirical observation

$$(t, -t_3, -t_0)_L = \frac{1}{6}(N, w, r + 1) \quad (1.3)$$

where  $t_0$  is the electroweak  $U(1)$  hypercharge. Equation (1.3) relates the four left chiral families of elementary fermions described by  $(\frac{1}{2}, t_3, t_0)_L$  to the four quantum trefoils described by  $(3, w, r + 1)$  and is supported by the row to row proportionality in Table 1 as expressed in equation (1.3).

**Table 1:** Empirical Support for (1.3)

	$(f_1, f_2, f_3)$	$t$	$t_3$	$t_0$	$D_{\frac{w}{2} \frac{r+1}{2}}^{N/2}$	$N$	$w$	$r$	$r+1$
leptons	$(e, \mu, \tau)_L$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$D_{\frac{3}{2} \frac{3}{2}}^{3/2}$	3	3	2	3
	$(\nu_e, \nu_\mu, \nu_\tau)_L$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$D_{-\frac{3}{2} \frac{3}{2}}^{3/2}$	3	-3	2	3
quarks	$(d, s, b)_L$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$D_{\frac{3}{2} - \frac{1}{2}}^{3/2}$	3	3	-2	-1
	$(u, c, t)_L$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$	$D_{-\frac{3}{2} - \frac{1}{2}}^{3/2}$	3	-3	-2	-1

Only for the particular row-to-row correspondences shown in Table 1 does (1.3) hold, i.e., each of the four families of fermions labelled by  $(t_3, t_0)$  is *uniquely correlated with a specific  $(w, r)$  trefoil, and therefore with a specific  $D_{mm'}^{3/2}$  quantum knot.*

Note also that with this same correspondence all the leptons (regarding the neutrinos as uncharged leptons) correspond to quantum trefoils with positive knot helicity ( $r = 2$ ) while the quarks correspond to quantum trefoils of opposite knot helicity ( $r = -2$ ).

By (1.2) and (1.3) one also has

$$(j, m, m') = 3(t, -t_3, -t_0)_L \quad (1.4)$$

for the left chiral fields and quantum trefoils. Quantum trefoils are therefore jointly defined by the topological condition (1.2) and the empirical condition (1.4). Both (1.2) and (1.4) relate the quantum trefoils to the left chiral field components of the fermionic field operators of the standard model. With the aid of Noether charges carried by  $D_{mm'}^j$  knots we shall explore the extension of (1.4) to the more general case where  $j \neq 3/2$  and  $o \neq 1$ .

## 2 Noether Charges carried by $D_{mm'}^j$ Knots

The irreducible representations of  $\text{SLq}(2)$  may be expressed as follows<sup>(3)</sup>

$$D_{mm'}^j(q|a, b, c, d) = \sum_{\substack{\delta(n_a+n_b, n_+) \\ \delta(n_c+n_d, n_-)}} A_{mm'}^j(q|n_a, n_c) \delta(n_a + n_c, n'_+) a^{n_a} b^{n_b} c^{n_c} d^{n_d} \quad (2.1)$$

Here the arguments  $(a, b, c, d)$  satisfy the following algebra

$$\begin{aligned} ab &= qba & bd &= qdb & ad - qbc &= 1 & bc &= cb \\ ac &= qca & cd &= qdc & da - q_1cb &= 1 & q_1 &\equiv q^{-1} \end{aligned} \quad (2.2)$$

The numerical coefficients are

$$A_{mm'}^j(q|n_a, n_c) = \left[ \frac{\langle n'_+ \rangle_1 \langle n'_- \rangle_1}{\langle n_+ \rangle_1 \langle n_- \rangle_1} \right]^{\frac{1}{2}} \frac{\langle n_+ \rangle_1!}{\langle n_a \rangle_1! \langle n_b \rangle_1!} \frac{\langle n_- \rangle_1!}{\langle n_c \rangle_1! \langle n_d \rangle_1!} \quad (2.3)$$

where

$$\begin{aligned} n_{\pm} &= j \pm m \\ n'_{\pm} &= j \pm m' \end{aligned} \quad (2.4)$$

and

$$\begin{aligned} \langle n \rangle_q &= \frac{q^n - 1}{q - 1} \\ \langle n \rangle_1 &\equiv \langle n \rangle_{q_1} \end{aligned} \quad (2.5)$$

The sum (2.1) is taken over the positive integers  $n_a, n_b, n_c, n_d$  subject to the  $\delta$ -function constraints as shown.  $q$  is taken to be real.

$D_{mm'}^j$  is defined only up to the following gauge transformation on  $(a, b, c, d)$  that leaves the algebra (2.2) invariant:

$$\begin{aligned} a' &= e^{i\varphi_a} a & b' &= e^{i\varphi_b} b \\ d' &= e^{-i\varphi_a} d & c' &= e^{-i\varphi_b} c \end{aligned} \quad (2.6)$$

We shall also refer to the transformation described by (2.6) as  $U_a(1) \times U_b(1)$ .

The transformation (2.6),  $U_a(1) \times U_b(1)$ , on the  $(a, b, c, d)$  of  $\text{SLq}(2)$  induces on the  $D_{mm'}^j$  of  $\text{SLq}(2)$  the corresponding transformation<sup>(3)</sup>

$$\begin{aligned} D_{mm'}^j(a, b, c, d) &\rightarrow D_{mm'}^j(a', b', c', d') \\ &= e^{i(\varphi_a + \varphi_b)m} e^{i(\varphi_a - \varphi_b)m'} D_{mm'}^j(a, b, c, d) \\ &= U_m(1) \times U_{m'}(1) D_{mm'}^j(a, b, c, d) \end{aligned}$$

and by equation (1.1) on the field operators

$$\Psi_{mm'}^j \rightarrow U_m(1) \times U_{m'}(1) \Psi_{mm'}^j \quad (2.7)$$

For physical consistency any allowed field action must be invariant under (2.7) since (2.7) is induced by  $U_a \times U_b$  transformations that leave the defining algebra (2.2) unchanged. There are then Noether charges associated with  $U_m$  and  $U_{m'}$  that may be described as writhe and rotation charges,  $Q_w$  and  $Q_r$ , since  $m = \frac{w}{2}$  and  $m' = \frac{1}{2}(r + o)$  for quantum knots.

For quantum trefoils we have set  $o = 1$ , and we now define their Noether charges:

$$Q_w \equiv -k_w m \quad \left( \equiv -k_w \frac{w}{2} \right) \quad (2.8)$$

$$Q_r \equiv -k_r m' \quad \left( \equiv -k_r \frac{1}{2}(r + 1) \right) \quad (2.9)$$

where  $k_w$  and  $k_r$  are undetermined constants with dimensions of electric charge. We assume that  $k_w = k_r = k$  is a universal constant with the same value for all trefoils.

In Table 2 we next compare the electric charges  $Q_e$  of the elementary fermions with the total Noether charges of the corresponding quantum trefoils. To construct and interpret this table we have again postulated that  $k_w = k_r = k$  is a universal constant with the same value for all trefoils. We then obtain the value of  $k$  by requiring that the total Noether charge,  $Q_w + Q_r$ , of the quantum trefoil  $D_{\frac{3}{2}\frac{3}{2}}^{3/2}$ , corresponding to the charged leptons of Table 1, satisfies

$$Q_w + Q_r \equiv Q_e \quad (2.10)$$

where  $Q_w$  and  $Q_r$  are the writhe and rotation charges, and  $Q_e$  is the electric charge of the corresponding family of elementary fermions, the charged leptons as shown in Table 2.

One sees that (2.10) holds not only for charged leptons, but also for neutrinos and for both up and down quarks if

$$k = \frac{e}{3} \quad (2.11)$$

**Table 2:** Electric Charges on Leptons, Quarks, and Quantum Trefoils

Standard Model					Quantum Trefoil Model				
$(f_1, f_2, f_3)$	$t$	$t_3$	$t_0$	$Q_e$	$(N, w, r)$	$D^{\frac{N/2}{\frac{w}{2} \frac{r+1}{2}}}$	$Q_w$	$Q_r$	$Q_w + Q_r$
$(e, \mu, \tau)_L$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-e$	$(3, 3, 2)$	$D^{\frac{3/2}{\frac{3}{2} \frac{2}{2}}}$	$-k \left(\frac{3}{2}\right)$	$-k \left(\frac{3}{2}\right)$	$-3k$
$(\nu_e, \nu_\mu, \nu_\tau)_L$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$0$	$(3, -3, 2)$	$D^{\frac{3/2}{-\frac{3}{2} \frac{3}{2}}}$	$-k \left(-\frac{3}{2}\right)$	$-k \left(\frac{3}{2}\right)$	$0$
$(d, s, b)_L$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{1}{3}e$	$(3, 3, -2)$	$D^{\frac{3/2}{\frac{3}{2} - \frac{1}{2}}}$	$-k \left(\frac{3}{2}\right)$	$-k \left(-\frac{1}{2}\right)$	$-k$
$(u, c, t)_L$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{3}e$	$(3, -3, -2)$	$D^{\frac{3/2}{-\frac{3}{2} - \frac{1}{2}}}$	$-k \left(-\frac{3}{2}\right)$	$-k \left(-\frac{1}{2}\right)$	$2k$
$Q_e = e(t_3 + t_0)$					$Q_w = -k \frac{w}{2} \quad Q_r = -k \frac{r+1}{2}$				

and also that  $t_3$  and  $t_0$  measure the writhe and rotation charges respectively:

$$Q_w = et_3 \left( = -\frac{e}{3}m = -\frac{e}{6}w \right) \quad (2.12)$$

$$Q_r = et_0 \left( = -\frac{e}{3}m' = -\frac{e}{6}(r+1) \right) \quad (2.13)$$

Then (2.10) becomes by (2.12) and (2.13) an alternative statement of

$$Q_e = e(t_3 + t_0) \quad (2.14)$$

Also by (2.12) and (2.13)

$$Q_e = -\frac{e}{3}(m + m'), \quad (2.15)$$

or

$$Q_e = -\frac{e}{6}(w + r + 1). \quad (2.16)$$

for the quantum trefoils.

Then the electric charge is a measure of the writhe + rotation of the trefoil. The total electric charge in this way resembles the total angular momentum and total magnetic moment as a sum of two parts where the knot rotation corresponds to the orbital angular momentum and the magnetic moment and where the localized contribution of the writhe to the charge corresponds to the localized contribution of the spin to the angular momentum

and magnetic moment. In (2.16)  $o$  contributes a “zero-point charge.”

The total  $SL_q(2)$  charge sums the signed clockwise and counterclockwise turns that any current makes both at the crossings and in going once around the 2d-projected knot. In this way, the handedness of the knot determines its charge, so that handedness reduces charge to a geometrical concept similar to the way that curvature of space-time encodes mass and energy. This measure of charge, which is suggested by the leptons and quarks, goes to a deeper level than the electroweak isotopic measure that originated in the neutron-proton system.

As here defined, quantum knots carry the charge expressed as both (2.14) and (2.15). The  $(t_3, t_0)$  measures of charge are based on  $SU(2) \times U(1)$  while the  $(m, m')$  measures of charge are based on  $SL_q(2)$ . These two different measures are related at the  $j = \frac{3}{2}$  level by (1.4). We shall next attempt to extend these results beyond  $j = \frac{3}{2}$ , and in particular to  $j = \frac{1}{2}$ .

### 3 The Physical Interpretation of $D_{mm'}^j$

We may give physical meaning to the defining expression (2.1) for  $D_{mm'}^j$  by *interpreting the  $a, b, c, d$  as creation operators for fermionic preons*, since these are the four elements of the fundamental ( $j = \frac{1}{2}$ ) representation given by:

$$D_{mm'}^{1/2} = \begin{array}{c|cc} & & m' \\ & & \frac{1}{2} \quad -\frac{1}{2} \\ m & & \\ \hline & \frac{1}{2} & a \quad b \\ & -\frac{1}{2} & c \quad d \end{array} \quad (3.1)$$

*By (3.1) and (2.15) there is one charged preon,  $a$ , with charge  $-\frac{e}{3}$  and its antiparticle,  $d$ , and there is one neutral preon,  $b$ , with its antiparticle,  $c$ .*

By (1.2) the corresponding  $a, b, c, d$  classical realizations cannot be described as knots since they have only a single crossing. They can, however, be described as 2d-projections of twisted loops with  $N = 1$ ,  $w = \pm 1$  and  $r = 0$ . We may propose a physical meaning for these twisted loops by interpreting their quantum realizations  $\hat{\Psi}_{mm'}^{1/2} D_{mm'}^{1/2}$  by the standard quantum Lagrangian as flux tubes, and we may regard  $a, b, c, d$  as creation operators for either preonic particles or preonic 2d-projections of flux tubes, depending on whether we assume that they concentrate energy and momentum at a point or on a curve. We may

assume that the direction of flow in the flux tube defines its helicity.

Every  $D_{mm'}^j$ , as given in (2.1), being a polynomial in  $a, b, c, d$ , can be interpreted as a creation operator for creating a superposition of states, each state with  $n_a, n_b, n_c, n_d$  preons. The  $a, b, c, d$  population of each of these states is constrained by the triplet  $(j, m, m')$  that allows  $(n_a, n_b, n_c, n_d)$  to vary but fixes  $(t, t_3, t_0)$  and  $(N, w, r+o)$  according to (1.4) and (1.2).

It then turns out that the creation operators for the charged leptons,  $D_{\frac{3}{2}\frac{3}{2}}^{3/2}$ ; neutrinos,  $D_{-\frac{3}{2}\frac{3}{2}}^{3/2}$ ; down quarks,  $D_{\frac{3}{2}-\frac{1}{2}}^{3/2}$ ; and up quarks,  $D_{-\frac{3}{2}-\frac{1}{2}}^{3/2}$ , as empirically required by Tables 1 and 2, are represented by (2.1) as the following monomials

$$D_{\frac{3}{2}\frac{3}{2}}^{3/2} \sim a^3, \quad D_{-\frac{3}{2}\frac{3}{2}}^{3/2} \sim c^3, \quad D_{\frac{3}{2}-\frac{1}{2}}^{3/2} \sim ab^2, \quad D_{-\frac{3}{2}-\frac{1}{2}}^{3/2} \sim cd^2 \quad (3.2)$$

implying that charged leptons and neutrinos are composed of three  $a$ -preons and three  $c$ -preons, respectively, while the down quarks are composed of one  $a$ - and two  $b$ -preons, and the up quarks are composed of one  $c$ - and two  $d$ -preons. Both (3.1), with (2.15), and (3.2) are in agreement with the Harari-Shupe model of quarks, and with the experimental evidence on which their model is constructed.<sup>(4)(5)</sup>

To achieve the required  $U_a(1) \times U_b(1)$  invariance of the knotted Lagrangian (and the associated conservation of  $t_3$  and  $t_0$ , or equivalently of the writhe and rotation charge), it is necessary to impose (1.2) and (1.4) on the knotted vector bosons by which the knotted fermions interact as well as on the knotted fermions themselves. For these electroweak vector fields we assume the  $t = 1$  of the standard model and therefore  $j = 3$  and  $N = 6$  since  $(j, m, m') = 3(1, -t_3, -t_0)$ , in accord with (1.4) and (1.2) and as shown in Table 3.

**Table 3:** Electroweak Vectors ( $j = 3$ )

	$Q$	$t$	$t_3$	$t_0$	$D_{-3t_3-3t_0}^{3t}$
$W^+$	$e$	1	1	0	$D_{-3,0}^3 \sim c^3 d^3$
$W^-$	$-e$	1	-1	0	$D_{3,0}^3 \sim a^3 b^3$
$W^3$	0	1	0	0	$D_{0,0}^3 \sim f_3(bc)$

The charged  $W_\mu^+$  and  $W_\mu^-$  are six preon monomials. The neutral vector  $W_\mu^3$  is the superposition of four states of six preons given by

$$D_{00}^3 = A(0, 3)b^3c^3 + A(1, 2)ab^2c^2d + A(2, 1)a^2bcd^2 + A(3, 0)a^3d^3 \quad (3.3)$$



according to (2.1) which is reducible by the algebra (2.2) to a function of the neutral operator  $bc$ . It may be assumed that the  $W^0$  neutral vector coupled to the hypercharge is unknotted.

The previous considerations are based on electroweak physics. To describe the strong interactions it is necessary according to the standard model to introduce  $SU(3)$ . In the  $SLq(2)$  electroweak model, as here described, the need for the additional  $SU(3)$  symmetry appears already at the level of the charged leptons and neutrinos since they are presented in the  $SLq(2)$  model at the electroweak level as  $a^3$  and  $c^3$ , respectively. Then the simple way to protect the Pauli principle is to replace  $(a, c)$  by  $(a_i, c_i)$  and replace

$$\text{charged leptons } a^3 \text{ by } \varepsilon^{ijk} a_i a_j a_k$$

$$\text{neutrinos } c^3 \text{ by } \varepsilon^{ijk} c_i c_j c_k$$

where  $a_i$  and  $c_i$  provide a basis for the fundamental representation of  $SU(3)$ . Then the charged leptons and neutrinos are color singlets. If the  $b$  and  $d$  preons are also color singlets, then down quarks  $a_i b^2$  and up quarks  $c_i d^2$  provide a basis for the fundamental representation of  $SU(3)$ , as required by the standard model.<sup>(6)</sup>

We do not depart from  $SLq(2)$  in the above way of introducing  $SU(3)$ . If one instead goes over to  $SUq(2)$ , where  $\bar{a} = d$  and  $\bar{c} = -q_1 b$ , we may make use of the two complex representations  $3$  and  $\bar{3}$  of  $SU(3)$ , by assigning  $a_i$  and  $c_i$  to the  $\bar{3}$  and  $b^i$  and  $d^i$  to the  $3$  representation.<sup>(7)</sup>

The leptons and quarks, which are associated in Table 1 with opposite knot helicity ( $r$ ), are  $SU(3)$  singlets and  $SU(3)$  triplets, respectively, for gluon interactions. Likewise the  $SU(3)$  triplets,  $a_i$  and  $c_i$ , have positive preon helicity ( $\tilde{r} = +1$ ), while the  $SU(3)$  singlets,  $b$  and  $d$ , have opposite preon helicity ( $\tilde{r} = -1$ ). The association of the  $SU(3)$  representation with knot and preon helicity repeats the similar association of  $SU(2)$  doublets and  $SU(2)$  singlets with left and right chirality, respectively, for electroweak interactions.

## 4 Complementarity<sup>(3)</sup>

The representation of  $D_{mm'}^j$  as a function of  $(a, b, c, d)$  and  $(n_a, n_b, n_c, n_d)$  by Equation (2.1) implies the following constraints on the exponents:

$$n_a + n_b + n_c + n_d = 2j \tag{4.1}$$

$$n_a + n_b - n_c - n_d = 2m \tag{4.2}$$

$$n_a - n_b + n_c - n_d = 2m' \tag{4.3}$$

The two relations giving physical meaning to  $D_{mm'}^j$ , namely (1.2) and (1.4):

$$(j, m, m') = \frac{1}{2}(N, w, r + o) \quad (4.4)$$

and

$$(j, m, m') = 3(t, -t_3, -t_0)_L \quad (4.5)$$

imply two different interpretations of the relations (4.1)-(4.3). By (4.4) one has

$$N = n_a + n_b + n_c + n_d \quad (4.6)$$

$$w = n_a + n_b - n_c - n_d \quad (4.7)$$

$$\tilde{r} \equiv r + o = n_a - n_b + n_c - n_d \quad (4.8)$$

In (4.8), where  $\tilde{r} \equiv r + o$  and  $o$  is the parity index,  $\tilde{r}$  has been termed “the quantum rotation,” and  $o$  the “zero-point rotation.”

By (4.5) one has

$$t = \frac{1}{6}(n_a + n_b + n_c + n_d) \quad (4.9)$$

$$t_3 = -\frac{1}{6}(n_a + n_b - n_c - n_d) \quad (4.10)$$

$$t_0 = -\frac{1}{6}(n_a - n_b + n_c - n_d) \quad (4.11)$$

These relations hold for all representations allowed by the model. For the elements of the fundamental representation they imply the Tables 4 and 5 describing the fermionic preons. Tables 4 and 5 are related by (1.3) as satisfied by preons in the following equation:

$$(t_p, -t_{3_p}, -t_{0_p})_L = \frac{1}{6}(N_p, w_p, r_p + 1) \quad p = a, b, c, d \quad (1.3)_p$$

Equation 4.5 describes the electroweak indices  $(t, t_3, t_0)$  if  $j = \frac{3}{2}$ ; in general, however, in this model including  $j = \frac{1}{2}$  and Table 5, the  $(t, t_3, t_0)$  should be understood as code for  $(j, m, m')$ , i.e. as indices for SLq(2), not for SU(2)×U(1). Then the index  $t_3$  in (4.5) measures writhe charge and  $t_0$  measures rotation hypercharge as Noether charges following from the  $U_a \times U_b$  invariance of the knot Lagrangian. Then  $t$  no longer needs to be integral or half-integral. The “SLq(2) knot charge” defines charge more naturally in the knot model than electroweak isotopic charge with which it agrees at  $j = \frac{3}{2}$  since the SU(2)×U(1) measure has its origins in the proton-neutron isotopic spin while the SLq(2) measure is suggested by quark-lepton charges. At the  $j = \frac{3}{2}$  level the SU(2)×U(1) measure requires the assumption of fractional charges for the quarks and the SLq(2) measure requires at the

$j = \frac{1}{2}$  level the replacement of the fundamental charge ( $e$ ) for charged leptons by a new fundamental charge ( $e/3$ ) for charged preons. The  $\text{SLq}(2)$ , or  $(j, m, m')$  measure, has a direct physical interpretation since  $(j, m, m') = \frac{1}{2}(N, w, r + o)$ , where  $2j$  is the number of preonic sources, while  $2m$  and  $2m'$  respectively measure the numbers of writhe and rotation sources of preonic charge.<sup>(3)</sup>

If neutral unknotted flux tubes predated the particles, and the particles were initially formed by the knotting of the neutral flux tubes, then the simplest particles that could have formed must have had 3 crossings and by (4.6) three preons. *The electric charge of the resultant trefoil or of any composite of preons would then be a measure of the handedness generated by the knotting of the original unknotted flux loop.*

**Table 4:** Elements of  $j = \frac{1}{2}$  Representation as Twisted Loops

$p$	$N_p$	$w_p$	$\tilde{r}_p$
$a$	1	1	1
$b$	1	1	-1
$c$	1	-1	1
$d$	1	-1	-1

**Table 5:** Elements of  $j = \frac{1}{2}$  Representation as Point Particles

$p$	$t_p$	$t_{3_p}$	$t_{0_p}$	$Q_p$
$a$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{e}{3}$
$b$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{6}$	0
$c$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	0
$d$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{e}{3}$

In Equations (4.6)-(4.8), the numerical coefficients may be replaced by  $(N_p, w_p, \tilde{r}_p)$  from Table 4 as follows:

$$N = \sum_p n_p N_p, \quad p = (a, b, c, d) \quad (4.12)$$

$$w = \sum_p n_p w_p \quad (4.13)$$

$$\tilde{r} = \sum_p n_p \tilde{r}_p \quad (4.14)$$

and in Equations (4.9)-(4.11), the numerical coefficients may be replaced by  $(t_p, t_{3_p}, t_{0_p})$

and Table 5 as follows:

$$t = \sum_p n_p t_p, \quad p = (a, b, c, d) \quad (4.15)$$

$$t_3 = \sum_p n_p t_{3p} \quad (4.16)$$

$$t_0 = \sum_p n_p t_{0p} \quad (4.17)$$

Since  $r = 0$  for preonic loops,  $o$  plays the role of a quantum rotation for preons:

$$\tilde{r}_p = o_p \quad p = (a, b, c, d) \quad (4.18)$$

For the elementary fermions presently observed,

$$\tilde{r} = r + 1. \quad (4.19)$$

We shall now regard  $D_{mm'}^j$  as the creation operator for a superposition of quantum states that may be described as either the 2d-projections of knotted fields  $(N, w, r)$  composed of 2d-projections of preonic flux lines according to (4.12)-(4.14) or as composite particles  $(t, t_3, t_0)$  composed of preonic particles according to (4.15)-(4.17). As formal algebraic relations (4.12)-(4.17) express properties of the higher representations as additive compositions of the fundamental representation.

*Equation (4.6) states that the total number of preons,  $N'$ , equals the number of crossings,  $N$ . Since we assume that the preons are fermions, the knot describes a fermion or a boson depending on whether the number of crossings is odd or even. Viewed as a knot, a fermion becomes a boson when the number of crossings is changed by attaching or removing a curl. This picture is consistent with the view of a curl as an opened preon loop. One may also assume that each counterclockwise or clockwise classical curl corresponds to a quantal preon creation operator or antipreon creation operator respectively.*

Since  $a$  and  $d$  are creation operators for particles and antiparticles with opposite charge and hypercharge, while  $b$  and  $c$  are neutral particles and antiparticles with opposite values of the hypercharge, we may introduce the preon numbers

$$\nu_a = n_a - n_d \quad (4.20)$$

$$\nu_b = n_b - n_c \quad (4.21)$$

Then (4.7) and (4.8) may be rewritten in terms of preon numbers as

$$\nu_a + \nu_b = w (= -6t_3) \quad (4.22)$$

$$\nu_a - \nu_b = \tilde{r} (= -6t_0) \quad (4.23)$$

By (4.22) and (4.23) the conservation of the preon numbers and of the charge and hypercharge is equivalent to the conservation of the writhe and rotation, which are topologically conserved at the 2d-classical level. In this respect, these quantum conservation laws correspond to the classical conservation laws.

One may view the symmetry of an elementary particle, defined by the representations of the  $\text{SLq}(2)$  algebra, in any of the following ways:

$$D_{mm'}^j = D_{-3t_3-3t_0}^{3t} = D_{\frac{w}{2}\frac{\tilde{r}}{2}}^{N/2} = \tilde{D}_{\nu_a\nu_b}^{N'}, \quad (4.24)$$

where  $N'$  is the total number of preons.

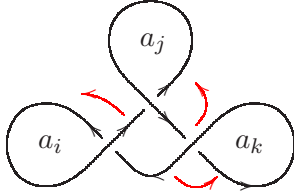

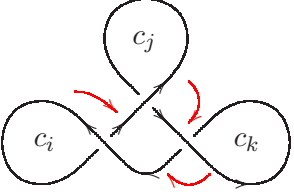

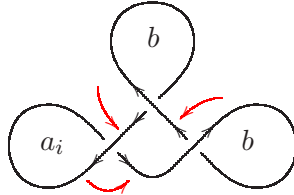
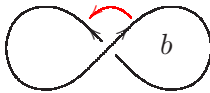
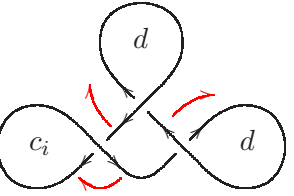
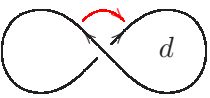
The point particle  $(N', \nu_a, \nu_b)$  representation and the flux loop  $(N, w, \tilde{r})$  complementary representation are related by

$$\tilde{D}_{\nu_a, \nu_b}^{N'} = \sum_{N, w, r} \delta(N', N) \delta(\nu_a + \nu_b, w) \delta(\nu_a - \nu_b, \tilde{r}) D_{\frac{w}{2}\frac{\tilde{r}}{2}}^{N/2} \quad (4.25)$$

The 2d-representation of the four classical trefoils as composed of three overlapping preon loops is shown in Figure 1. In interpreting Figure 1, note that the two lobes of all the preon loops make opposite contributions to the rotation,  $r$ , so that the total rotation of each preon loop vanishes. When the three  $a$ -preons and  $c$ -preons are combined to form charged leptons and neutrinos, respectively, each of the three labelled circuits is counterclockwise and contributes  $+1$  to the rotation while the single unlabeled shared (overlapping) circuit is clockwise and contributes  $-1$  to the rotation so that the total  $r$  for both charged leptons and neutrinos is  $+2$ . For quarks the three labelled loops contribute  $-1$  and the shared loop  $+1$  so that  $r = -2$ . It is the quantum rotation ( $\tilde{r}$ ), however, and not the classical rotation ( $r$ ) that satisfies (4.14).

**Figure 1:** Preonic Structure of Elementary Fermions

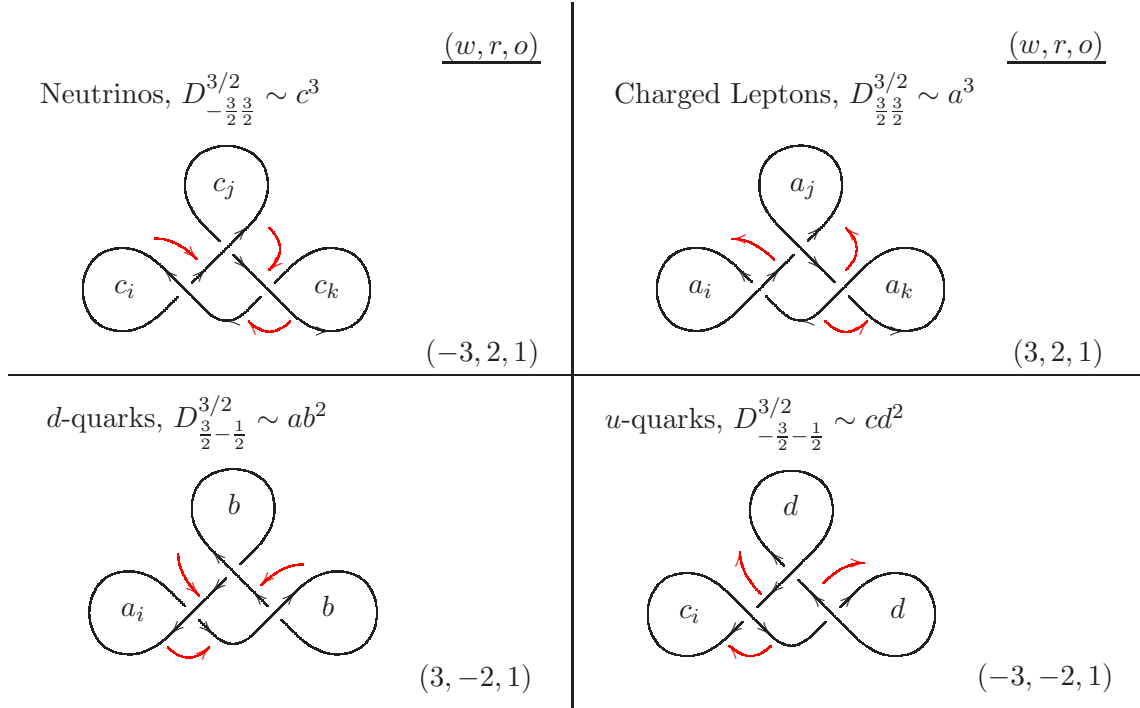
$$Q = -\frac{e}{6}(w + r + o), \text{ and } (j, m, m') = \frac{1}{2}(N, w, r + o)$$

<p>Charged Leptons, <math>D_{\frac{3}{2}\frac{3}{2}}^{3/2} \sim a^3</math></p> <p><u><math>(w, r, o)</math></u></p>  <p><math>(3, 2, 1)</math></p>	<p><u><math>(w, r, o)</math></u></p> <p><math>a</math>-preons, <math>D_{\frac{1}{2}\frac{1}{2}}^{1/2}</math></p>  <p><math>(1, 0, 1)</math></p>
<p>Neutrinos, <math>D_{-\frac{3}{2}\frac{3}{2}}^{3/2} \sim c^3</math></p>  <p><math>(-3, 2, 1)</math></p>	<p><math>c</math>-preons, <math>D_{-\frac{1}{2}\frac{1}{2}}^{1/2}</math></p>  <p><math>(-1, 0, 1)</math></p>
<p><math>d</math>-quarks, <math>D_{\frac{3}{2}-\frac{1}{2}}^{3/2} \sim ab^2</math></p>  <p><math>(3, -2, 1)</math></p>	<p><math>b</math>-preons, <math>D_{\frac{1}{2}-\frac{1}{2}}^{1/2}</math></p>  <p><math>(1, 0, -1)</math></p>
<p><math>u</math>-quarks, <math>D_{-\frac{3}{2}-\frac{1}{2}}^{3/2} \sim cd^2</math></p>  <p><math>(-3, -2, 1)</math></p>	<p><math>d</math>-preons, <math>D_{-\frac{1}{2}-\frac{1}{2}}^{1/2}</math></p>  <p><math>(-1, 0, -1)</math></p>

The sense of the writhe is shown by the red arrows and the sense of the rotation is shown by the black arrows.

In (4.25) the correspondence between classical and quantum knots introduced in (1.2) has here been promoted to a correspondence between fields and particles. The classical knots in this model provide a classical language for describing some aspects of the standard model without necessarily providing a specific physical realization of these knots. One goes further in (4.25) by postulating a special physical realization of the knots as follows. Since one may interpret the elements  $(a, b, c, d)$  of the  $SL_q(2)$  algebra as creation operators for either preonic particles or flux loops, the  $D_{mp}^j$  may be interpreted as a creation operator for a composite particle composed of either preonic particles or flux loops. *These two complementary views of the same particle may be reconciled as describing  $N$ -body systems bound by a knotted field having  $N$ -crossings as illustrated in Figure 2 for  $N = 3$ .* In the limit where the three outside lobes become infinitesimal compared to the central circuit, the resultant structure will resemble a three particle system tied together by a string. The  $j = 1$  representation does not play an explicit role in the picture just described. In a different physical interpretation of the algebra, the  $j = 1$  vector field binds the three  $j = \frac{1}{2}$  preons to form the  $j = \frac{3}{2}$  elementary fermions.

**Figure 2:** Leptons and Quarks Pictured as Three Preons Bound by a Trefoil Field



The preons conjectured to be present at the crossings are not shown in these figures.

The physical models suggested by Fig. 2 may be further studied in the context of gravitational and gluon binding with the aid of the preon Lagrangian given in reference 3.

On the other hand, in an alternative interpretation of complementarity, the hypothetical preons conjectured to be present in Figure 2 carry no independent degrees of freedom and may simply describe concentrations of energy, momentum, and charge at the crossings of the flux tube. In this interpretation of complementarity,  $(t, t_3, t_0)$  and  $(N, w, \tilde{r})$  are just two ways of describing the same quantum trefoil of field. In this picture the preons are bound, i.e. they do not appear as free particles. This view of the elementary particles as non-singular lumps of field has also been described as a unitary field theory.

If  $j = 0$ , the indices of the quantum knot are

$$(j, m, m') = (0, 0, 0) \quad (4.26)$$

and by the rules (4.4) and (4.5) for interpreting the knot indices on the left chiral fields

$$(N, w, \tilde{r}) = (0, 0, 0) \quad \text{by (4.4)} \quad (4.27)$$

$$(t, -t_3, -t_0) = (0, 0, 0) \quad \text{by (4.5)} \quad (4.28)$$

Then by (4.28) the  $j = 0$  quantum states have no electroweak interactions and by (4.27) they correspond to classical loops with no crossings ( $N = 0$ ) just as preon states correspond to classical twisted loops with one crossing. It is possible that these hypothetical quantum states are realized as (electroweak non-interacting) loops of field flux with  $\tilde{r} = 0$ , and  $r = \pm 1$ ,  $o = \mp 1$  i.e. with the topological rotation  $r = \pm 1$ .

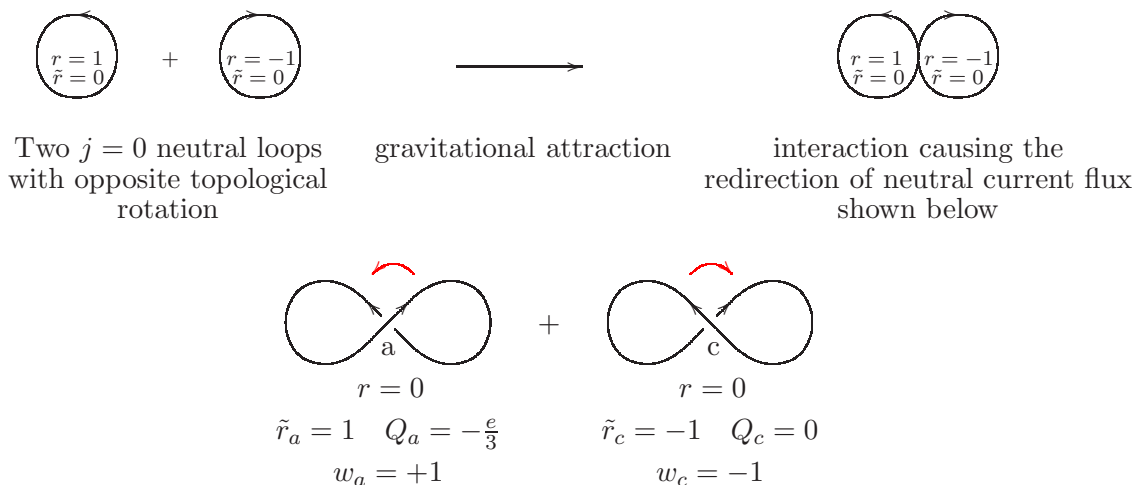
The  $(N, w, r)$  knot indices were first introduced to define the state space of the model by restricting the kinematics but without specifying the knot. If, as we are assuming, the leptons and quarks with  $j = \frac{3}{2}$  correspond to 2d projections of knots with three crossings, and if the heavier preons with  $j = \frac{1}{2}$  correspond to 2d projections of twisted loops with one crossing, then if the  $j = 0$  states correspond to 2d projections of simple loops, one might conjecture that these particles with no electroweak interactions are smaller and heavier than the preons, and are among the candidates for “dark matter.” If the  $j = 0$  particles predated the  $j = \frac{1}{2}$  preons, one may refer to the  $j = 0$  particles as yons as suggested by the term “ylem” for primordial matter.<sup>(8)</sup>

One may speculate about an earlier universe before leptons and quarks had appeared when there was no charge, and when energy, momentum, and angular momentum existed only in the  $\text{SLq}(2)$   $j = 0$  neutral state as simple loop currents of energy, momentum, and angular momentum. Then the gravitational attraction would bring some pairs of opposing loops close enough to permit the transition from two  $j = 0$  loops to two opposing  $j = \frac{1}{2}$  twisted loops. A possible geometric scenario for the transformation of two simple loops of current (yons) with opposite rotations into two  $j = \frac{1}{2}$  twisted loops of current (preons) is shown in Fig. 3. To implement this scenario one would expect to go beyond the electroweak



dynamics. Without attempting to do this, one notes according to Fig. 3 that the fusion of two yons may result in a doublet of twisted loops similar to the Higgs doublet which is independently required to be a  $SL_q(2)$  singlet ( $j = 0$ ) and a  $SU(2)$  charge doublet ( $t = \frac{1}{2}$ ) by the mass term of the Lagrangian described in reference 3. Since the Higgs mass is also the inertial mass, one expects a fundamental connection with the gravitational field at this point.

**Figure 3:** Creation of Preons as Twisted Loops



The topological rotation of the twisted loop is  $r = 0$  but by (4.18) each preon has a non-vanishing quantum rotation  $\tilde{r}_p = o_p$  determined by the odd integer that is required by the difference in parity between the writhe and the topological rotation. The relation of each of the four preons,  $a, b, c, d$ , to the twisted loop is shown in Table 4 where the degeneracy between  $a$  and  $b$  and between  $c$  and  $d$  is lifted by  $\tilde{r}_p$ , and in Table 5 is lifted by  $Q_e$  showing that  $\begin{pmatrix} c \\ a \end{pmatrix}$  and  $\begin{pmatrix} d \\ b \end{pmatrix}$  are charge doublets, as assumed in reference 3, and illustrated for  $\begin{pmatrix} c \\ a \end{pmatrix}$  in Figure 3.

If at an early time, only a fraction of the initial gas of quantum loops is converted to preons which in turn form a still smaller number of leptons and quarks, then most of the mass and energy of the universe would at the present time still reside in the dark loops while charge and current and visible mass would be confined to structures composed of leptons and quarks.

## 5 A Possible Interpretation of $q$ in the SLq(2) Model<sup>(3)</sup>

In the SLq(2) extension of the standard model, as so far presented here,  $q$  is regarded as a deformation parameter without a physical interpretation. Since the only physical coupling constants appearing explicitly in this model are electroweak, it is necessary to assign SU(3) indices to the preons in order to go beyond electroweak physics but the gluon couplings are then only implicit and are not completely introduced. It may be possible, however, to describe both the electroweak and the gluon couplings by interpreting  $q$  as a physical parameter. To explore this possibility one may present SLq(2) in terms of the following 2 parameter matrix:

$$\varepsilon_q = \begin{pmatrix} 0 & \alpha_2 \\ -\alpha_1 & 0 \end{pmatrix} \quad (5.1)$$

invariant under SLq(2) as follows:

$$T\varepsilon_q T^t = T^t \varepsilon_q T = \varepsilon_q \quad (5.2)$$

where  $t$  means transpose and where

$$T = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad (5.3)$$

Then (5.2) with (5.3) generates the SLq(2) algebra (2.2) as follows:

$$\begin{aligned} ab &= qba & bd &= qdb & ad - qbc &= 1 & bc &= cb \\ ac &= qca & cd &= qdc & da - q_1 cb &= 1 & q_1 &\equiv q^{-1} \end{aligned} \quad (5.4)$$

where

$$q = \frac{\alpha_1}{\alpha_2} \quad (5.5)$$

If one also requires

$$\det \varepsilon_q = 1 \quad (5.6)$$

then

$$\alpha_1 \alpha_2 = 1 \quad (5.7)$$

and  $\varepsilon_q$  underlies the structure of the Kauffman knot polynomial.<sup>(3)</sup>

Using this same algebra to describe the field theory of a particle that carries two different charges,  $e$  and  $g$ , we interpret the invariant matrix,  $\varepsilon_q$ , as a matrix coupling by setting

$$(\alpha_2, \alpha_1) \text{ or } (\alpha_1, \alpha_2) = \left( \frac{e}{\sqrt{\hbar c}}, \frac{g}{\sqrt{\hbar c}} \right) \quad (5.8)$$

where  $\alpha_1$  and  $\alpha_2$  are dimensionless and  $e$  and  $g$  have the dimensions of an electric charge. Then by (5.5)

$$q = \frac{e}{g} \text{ or } \frac{g}{e} \quad (5.9)$$

and if (5.6) is also imposed,<sup>(3)</sup>

$$eg = \hbar c \quad (5.10)$$

Here  $e$ ,  $g$ , and  $q$  are running coupling constants and  $e$  and  $g$  may be normalized to agree with experiment at hadronic energies. If  $e$  increases with energy and  $g$  decreases with energy according to asymptotic freedom,  $q$  may become very large or very small at the high energies where the interaction and mass terms become relevant for fixing the three particle bound states representing charged leptons, neutrinos and quarks. Although there is currently no experimental data suggesting the interpretation of  $q$  as a particular function of an  $e$  and a  $g$ , such a relation could be explored since  $e$ ,  $g$ , and  $q$  can be independently measured.

In this application of the  $SL_q(2)$  algebra the  $\varepsilon_q$  in (5.1), previously defining the knot polynomial, becomes a preon coupling matrix while the  $T$  in (5.3) becomes a creation matrix for  $a, b, c, d$  preons, and the  $T^t \varepsilon_q T$  in (5.2) may be read as an invariant operator that inserts form factors in the standard model.

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